1 a i $\overrightarrow{OR} = \frac{4}{5}\overrightarrow{OP}$
$= \frac{4}{5}\boldsymbol{p}$

ii $\overrightarrow{RP} = \frac{1}{5}\overrightarrow{OP}$
$= \frac{1}{5}\boldsymbol{p}$

iii $\overrightarrow{PO} = -\boldsymbol{p}$

iv $\overrightarrow{PS} = \frac{1}{5}\overrightarrow{PQ}$
$= \frac{1}{5}(\boldsymbol{q} - \boldsymbol{p})$

v $\overrightarrow{RS} = \overrightarrow{RP} + \overrightarrow{PS}$
$= \frac{1}{5}\boldsymbol{p} + \frac{1}{5}(\boldsymbol{q} - \boldsymbol{p})$
$= \frac{1}{5}\boldsymbol{q}$

b They are parallel (and $OQ = 5RS$).

c A trapezium (one pair of parallel lines).

d The area of triangle POQ is 25 times the area of $PRS = 125\text{cm}^2$.
$\therefore$ area of $ORSQ = 125 - 5$
$= 120 \text{ cm}^2$

2 a i $AP = \frac{2}{3}AB$ and $CQ = \frac{6}{7}CB$.
$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$
$= \overrightarrow{OA} + \frac{2}{3}\overrightarrow{AB}$
$= \boldsymbol{a} + \frac{2}{3}(\boldsymbol{b} - \boldsymbol{a})$
$= \frac{1}{3}\boldsymbol{a} + \frac{2}{3}\boldsymbol{b}$

ii $\overrightarrow{OQ} = \overrightarrow{OC} + \overrightarrow{CQ}$
$= \overrightarrow{OC} + \frac{6}{7}\overrightarrow{CB}$
$= k\boldsymbol{a} + \frac{6}{7}(\boldsymbol{b} - k\boldsymbol{a})$
$= \frac{k}{7}\boldsymbol{a} + \frac{6}{7}\boldsymbol{b}$

b i OPQ is a straight line if $OP = nOQ$.
$\frac{1}{3}\boldsymbol{a} + \frac{2}{3}\boldsymbol{b} = n\left(\frac{k}{7}\boldsymbol{a} + \frac{6}{7}\boldsymbol{b}\right)$
$= \frac{nk}{7}\boldsymbol{a} + \frac{6n}{7}\boldsymbol{b}$
$\frac{2}{3} = \frac{6n}{7}$
$n = \frac{14}{18} = \frac{7}{9}$

$$\frac{1}{3}\boldsymbol{a}+\frac{2}{3}\boldsymbol{b}=\frac{7}{9}\left(\frac{k}{7}a+\frac{6}{7}b\right)$$
$$=\frac{k}{9}\boldsymbol{a}+\frac{2}{3}\boldsymbol{b}$$
$$\frac{k}{9}=\frac{1}{3}$$
$$k=3$$

ii From part i

$$\overrightarrow{OP}=\frac{7}{9}\overrightarrow{OQ}$$
$$=\frac{7}{9}(OP+PQ)$$
$$=\frac{7}{9}OP+\frac{7}{9}PQ$$
$$\frac{2}{9}OP=\frac{7}{9}PQ$$
$$2OP=7PQ$$
$$\frac{OP}{PQ}=\frac{7}{2}$$

c

$$\overrightarrow{BC}=\overrightarrow{BO}+\overrightarrow{OC}$$
$$=-\boldsymbol{b}+k\boldsymbol{a}$$
$$=3\boldsymbol{a}-\boldsymbol{b},\text{since } k=3$$
$$\overrightarrow{PR}=\overrightarrow{PO}+\overrightarrow{OR}$$
$$=-\frac{1}{3}\boldsymbol{a}-\frac{2}{3}\boldsymbol{b}+\frac{7}{3}\boldsymbol{a}$$
$$=2\boldsymbol{a}-\frac{2}{3}\boldsymbol{b}$$
$$=\frac{2}{3}(3\boldsymbol{a}-\boldsymbol{b})$$
$$=\frac{2}{3}\overrightarrow{BC}$$

Hence PR is parallel to BC

3 a i

$$\overrightarrow{OD}=\frac{1}{3}\overrightarrow{OB}$$
$$=\frac{1}{3}(6\boldsymbol{i}-1.5\boldsymbol{j})$$
$$=2\boldsymbol{i}-0.5\boldsymbol{j}$$
$$\overrightarrow{AB}=3\boldsymbol{i}-6\boldsymbol{j}$$
$$\overrightarrow{AE}=\frac{1}{4}(3\boldsymbol{i}-5\boldsymbol{j})$$
$$=-0.75\boldsymbol{i}-1.25\boldsymbol{j}$$
$$\overrightarrow{OE}=\overrightarrow{OA}+\overrightarrow{AE}$$
$$=3\boldsymbol{i}+3.5\boldsymbol{j}+0.75\boldsymbol{i}-1.25\boldsymbol{j}$$
$$=3.75\boldsymbol{i}+2.25\boldsymbol{j}$$
$$=\frac{15}{4}\boldsymbol{i}+\frac{9}{4}\boldsymbol{j}$$

ii

$$\overrightarrow{ED}=2\boldsymbol{i}-0.5\boldsymbol{j}-\left(\frac{15}{4}\boldsymbol{i}+\frac{9}{4}\boldsymbol{j}\right)$$
$$=-\frac{6}{4}\boldsymbol{i}-\frac{11}{4}\boldsymbol{j}$$

$$|\overrightarrow{ED}| = \sqrt{\left(\frac{7}{4}\right)^2 + \left(\frac{11}{4}\right)^2}$$
$$= \sqrt{\frac{49+121}{16}}$$
$$= \sqrt{\frac{170}{16}}$$
$$= \frac{\sqrt{170}}{4}$$

b i $\overrightarrow{OX} = \frac{15p}{4}\boldsymbol{i} + \frac{9p}{4}\boldsymbol{j}$

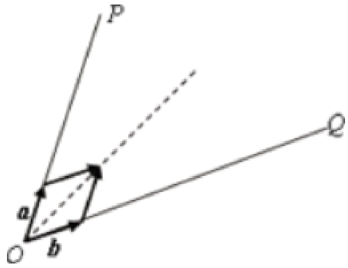
ii $\overrightarrow{AD} = 2\boldsymbol{i} - 0.5\boldsymbol{j} - (3\boldsymbol{i} + 3.5\boldsymbol{j})$
$$= -\boldsymbol{i} - 4\boldsymbol{j}$$
$$\overrightarrow{XD} = -q\boldsymbol{i} - 4q\boldsymbol{j}$$
$$\overrightarrow{OD} = \overrightarrow{OX} + \overrightarrow{OD}$$
$$\overrightarrow{OX} = \overrightarrow{OD} - \overrightarrow{XD}$$
$$= 2\boldsymbol{i} = 0.5\boldsymbol{j} - (-q\boldsymbol{i} - 4q\boldsymbol{j})$$
$$= (q+2)\boldsymbol{i} + (4q - 0.5)\boldsymbol{j}$$

c $(q+2)\boldsymbol{i} + (4q-0.5)\boldsymbol{j} = \frac{15p}{4}\boldsymbol{i} + \frac{9p}{4}\boldsymbol{j}$
$$q + 2 = \frac{15p}{4}$$
$$4q + 8 = 15p \quad (1)$$
$$4q - 0.5 = \frac{9p}{4} \quad (2)$$
$$(1) - (2):\ 8.5 = \frac{51p}{4}$$
$$p = \frac{8.5 \times 4}{51}$$
$$= \frac{2}{3}$$
$$q + 2 = \frac{15p}{4}$$
$$= \frac{10}{4} = \frac{5}{2}$$
$$q = \frac{1}{2}$$

4 a $\overrightarrow{PQ} = \boldsymbol{q} - \boldsymbol{p}$
$$= \overrightarrow{PM} + \overrightarrow{MQ}$$
$$\overrightarrow{MQ} = \frac{\beta}{\alpha}\overrightarrow{PM}$$
$$\therefore \overrightarrow{PQ} = \overrightarrow{PM} + \frac{\beta}{\alpha}\overrightarrow{PM}$$
$$= \frac{\alpha+\beta}{\alpha}\overrightarrow{PM}$$
$$\overrightarrow{PM} = \frac{\alpha}{\alpha+\beta}\overrightarrow{PQ}$$
$$\overrightarrow{OM} = \overrightarrow{OP} + \overrightarrow{PM}$$
$$= \boldsymbol{p} + \frac{\alpha}{\alpha + \boldsymbol{ba}}(\boldsymbol{q} - \boldsymbol{p})$$

$$= \frac{\alpha+\beta}{\alpha+\beta}\boldsymbol{p} + \frac{\alpha}{\alpha+ba}(\boldsymbol{q}-\boldsymbol{p})$$
$$= \frac{\alpha+\beta-\alpha}{\alpha+\beta}\boldsymbol{p} + \frac{\alpha}{\alpha+\beta}\boldsymbol{q}$$
$$= \frac{\beta\boldsymbol{p}+\alpha\boldsymbol{q}}{\alpha+\beta}$$

b **i**



It can be seen from the parallelogram formed by adding $\boldsymbol{a}$ and $\boldsymbol{b}$ that $\boldsymbol{a}+\boldsymbol{b}$ willlie on the bisector of angle POQ.

Hence any multiple, $\lambda(\boldsymbol{a}+\boldsymbol{b})$, will also lie on this bisector.

ii If $\boldsymbol{p}=k\boldsymbol{a}$ and $\boldsymbol{q}=l\boldsymbol{b}$, then

$$\overrightarrow{OM} = \frac{\beta\boldsymbol{p}+\alpha\boldsymbol{q}}{\alpha+\beta}$$
$$= \frac{\beta k\boldsymbol{a}+\alpha l\boldsymbol{b}}{\boldsymbol{a}n+\beta}$$

If M is the bisector of $\angle POQ$,

$$OM = \lambda\boldsymbol{a}+\lambda\boldsymbol{b}$$
$$\therefore\ \alpha l = \beta k$$

Divide both sides by βl:

$$\frac{\alpha}{\beta} = \frac{k}{l}$$

5 Let $OABC$ be a rhombus.

Let $\overrightarrow{OA} = \backslash\mathrm{bmit}a$ and $\overrightarrow{OC} = \backslash\mathrm{bmit}c$

We note that $|\backslash\mathrm{bmit}a| = |\backslash\mathrm{bmit}c|$

a **i** $\overrightarrow{AB} = \backslash\mathrm{bmit}c$

ii $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \backslash\mathrm{bmit}a + \backslash\mathrm{bmit}c$

iii $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = -\backslash\mathrm{bmit}a + \backslash\mathrm{bmit}c$

b $\overrightarrow{OB}\cdot\overrightarrow{AC} = (\backslash\mathrm{bmit}a + \backslash\mathrm{bmit}c)\cdot(-\backslash\mathrm{bmit}a + \backslash\mathrm{bmit}c)$

$$= -\backslash\mathrm{bmit}a\cdot\backslash\mathrm{bmit}a + \backslash\mathrm{bmit}c\cdot\backslash\mathrm{bmit}c$$
$$= -|\backslash\mathrm{bmit}a|^2 + |\backslash\mathrm{bmit}c|^2$$
$$= 0$$

c $\overrightarrow{OB}\cdot\overrightarrow{AC} = 0$ implies

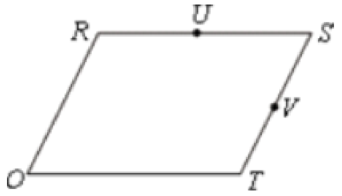
$-|\backslash\mathrm{bmit}a|^2 + |\backslash\mathrm{bmit}c|^2 = 0$

That is $|\backslash\mathrm{bmit}a| = |\backslash\mathrm{bmit}c|$

The parallelogram is a rhombus.
Conversely if the parallelogram is a rhombus, $|\backslash\mathrm{bmit}a| = |\backslash\mathrm{bmit}c|$.

Hence $\overrightarrow{OB}\cdot\overrightarrow{AC} = 0$

6



a $\boldsymbol{s} = \overrightarrow{OS}$

$= \overrightarrow{OR} + \overrightarrow{RS}$

$= \overrightarrow{OR} + \overrightarrow{OT}$

$= \boldsymbol{r} + \boldsymbol{t}$

b $\overrightarrow{ST} = \overrightarrow{OT} - \overrightarrow{OS}$

$= \boldsymbol{t} - \boldsymbol{s}$

$\boldsymbol{v} = \overrightarrow{OV}$

$= \overrightarrow{OS} + \overrightarrow{SV}$

$= \overrightarrow{OS} + \frac{1}{2}\overrightarrow{ST}$

$= \boldsymbol{s} - \frac{1}{2}(\boldsymbol{t} - \boldsymbol{s})$

$= \frac{1}{2}(\boldsymbol{s} + \boldsymbol{t})$

c Similarly:

$\boldsymbol{u} = \overrightarrow{OU}$

$= \overrightarrow{OS} + \overrightarrow{SU}$

$= \overrightarrow{OS} + \frac{1}{2}\overrightarrow{SR}$

$= \boldsymbol{s} - \frac{1}{2}(\boldsymbol{r} - \boldsymbol{s})$

$= \frac{1}{2}(\boldsymbol{s} + \boldsymbol{r})$

$\therefore\ \boldsymbol{u} + \boldsymbol{v} = \frac{1}{2}(\boldsymbol{s} + \boldsymbol{r}) + \frac{1}{2}(\boldsymbol{s} + \boldsymbol{t})$

$= \frac{1}{2}(2\boldsymbol{s} + \boldsymbol{r} + \boldsymbol{t})$

$2\boldsymbol{u} + 2\boldsymbol{v} = 2\boldsymbol{s} + \boldsymbol{r} + \boldsymbol{t}$

We may also express $\boldsymbol{u}$ as

$\boldsymbol{u} = \overrightarrow{OR} + \overrightarrow{RU}$

$= \overrightarrow{OR} + \frac{1}{2}\overrightarrow{RS}$

$= \overrightarrow{OR} + \frac{1}{2}\overrightarrow{OT}$

$= \boldsymbol{r} + \frac{1}{2}\boldsymbol{t}$

$\therefore\ \boldsymbol{u} + \boldsymbol{v} = 1 + \frac{1}{2}\boldsymbol{t} + \frac{1}{2}(\boldsymbol{s} + \boldsymbol{t})$

$= \frac{1}{2}(\boldsymbol{s} + 2\boldsymbol{r} + 2\boldsymbol{t})$

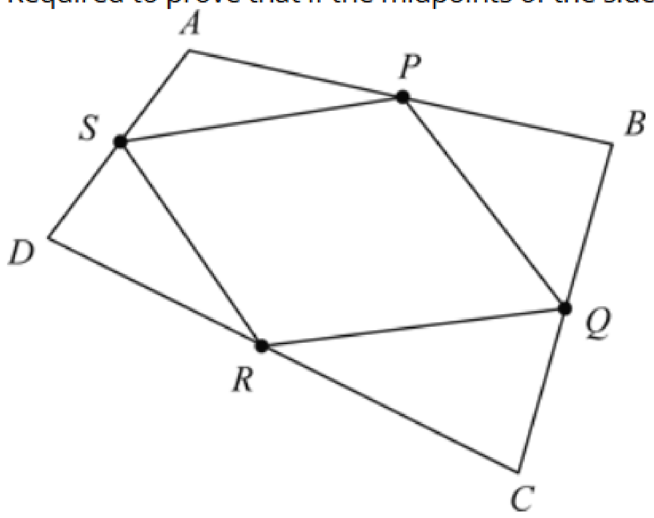
$2\boldsymbol{u} + 2\boldsymbol{v} = \boldsymbol{s} + 2\boldsymbol{r} + 2\boldsymbol{t}$

Add the two expressions for $2\boldsymbol{u} + 2\boldsymbol{v}$:

$4\boldsymbol{u} + 4\boldsymbol{v} = 3\boldsymbol{s} + 3\boldsymbol{r} + 3\boldsymbol{t}$

$= 3(\boldsymbol{s} + \boldsymbol{r} + \boldsymbol{t})$

7 Required to prove that if the midpoints of the sides of a quadrilateral are joined then a parallelogram if formed.



$ABCD$ is a quadrilateral. P, Q, R and S are the midpoints of the sides AB, BC, CD and DA respectively.

$$\overrightarrow{AS} = \frac{1}{2}\overrightarrow{AD}$$
$$\overrightarrow{AP} = \frac{1}{2}\overrightarrow{AB}$$
$$\begin{aligned}\overrightarrow{SP} &= \overrightarrow{AP} - \overrightarrow{AS}\\ &= \frac{1}{2}\overrightarrow{AB} - \frac{1}{2}\overrightarrow{AD}\\ &= \frac{1}{2}(\overrightarrow{AB} - \overrightarrow{AD})\\ &= \frac{1}{2}\overrightarrow{DB}\end{aligned}$$
$$\therefore \overrightarrow{SP} = \frac{1}{2}\overrightarrow{DB}$$

Similarly,

$$\overrightarrow{CR} = \frac{1}{2}\overrightarrow{CD}$$
$$\overrightarrow{CQ} = \frac{1}{2}\overrightarrow{CB}$$
$$\begin{aligned}\overrightarrow{RQ} &= \overrightarrow{RC} + \overrightarrow{CQ}\\ &= \frac{1}{2}\overrightarrow{CB} - \frac{1}{2}\overrightarrow{CD}\\ &= \frac{1}{2}(\overrightarrow{CB} - \overrightarrow{CD})2\\ &= \frac{1}{2}\overrightarrow{DB}\end{aligned}$$
$$\therefore \overrightarrow{RQ} = \frac{1}{2}\overrightarrow{DB}$$

Thus $\overrightarrow{SP} = \overrightarrow{RQ}$ meaning $SP \parallel RQ$ and $SP = RQ$

Hence $PQRS$ is a parallelogram.

8 Consider the square $OACB$.

Let $\overrightarrow{OA} = \backslash\mathrm{bmit}a$ and $\overrightarrow{OB} = \backslash\mathrm{bmit}b$

They are of equal magnitude. That is, $|\backslash\mathrm{bmit}a| = |\backslash\mathrm{bmit}b|$.

The diagonals are $\backslash\mathrm{bmit}a + \backslash\mathrm{bmit}b$ and $\backslash\mathrm{bmit}a - \backslash\mathrm{bmit}b$

$$\begin{aligned}|\backslash\mathrm{bmit}a + \backslash\mathrm{bmit}b|^2 &= (\backslash\mathrm{bmit}a + \backslash\mathrm{bmit}b) \cdot (\backslash\mathrm{bmit}a + \backslash\mathrm{bmit}b)\\ &= \backslash\mathrm{bmit}a \cdot \backslash\mathrm{bmit}a + 2\backslash\mathrm{bmit}a \cdot \backslash\mathrm{bmit}b + \backslash\mathrm{bmit}b \cdot \backslash\mathrm{bmit}b\\ &= |\backslash\mathrm{bmit}a|^2 + |\backslash\mathrm{bmit}b|^2\end{aligned}$$

$$\begin{aligned}|\backslash\mathrm{bmit}a - \backslash\mathrm{bmit}b|^2 &= (\backslash\mathrm{bmit}a - \backslash\mathrm{bmit}b)\cdot(\backslash\mathrm{bmit}a - \backslash\mathrm{bmit}b)\\ &= \backslash\mathrm{bmit}a\cdot\backslash\mathrm{bmit}a - 2\backslash\mathrm{bmit}a\cdot\backslash\mathrm{bmit}b + \backslash\mathrm{bmit}b\cdot\backslash\mathrm{bmit}b\\ &= |\backslash\mathrm{bmit}a|^2 + |\backslash\mathrm{bmit}b|^2\end{aligned}$$

The diagonals are of equal length

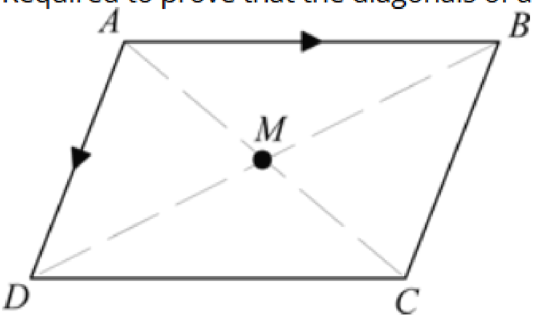
Let M be the midpoint of diagonal $\overrightarrow{OC}$. Then $\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OC} = \frac{1}{2}(\backslash\mathrm{bmit}a + \backslash\mathrm{bmit}b)$.

Let N be the midpoint of diagonal $\overrightarrow{BA}$.

Then $\overrightarrow{ON} = \overrightarrow{OB} + \frac{1}{2}(\backslash\mathrm{bmit}a - \backslash\mathrm{bmit}b) = \frac{1}{2}(\backslash\mathrm{bmit}a + \backslash\mathrm{bmit}b)$.

Therefore $M = N$. The diagonals bisect each other

9 Required to prove that the diagonals of a parallelogram bisect each other.



$ABCD$ is a parallelogram.

Let $\overrightarrow{AD} = \backslash\mathrm{bmit}a$

Let $\overrightarrow{AB} = \backslash\mathrm{bmit}b$

Let M be the midpoint of AC.

$$\begin{aligned}\overrightarrow{AC} &= \backslash\mathrm{bmit}b + \backslash\mathrm{bmit}a\\ \Rightarrow \overrightarrow{AM} &= \frac{1}{2}(\backslash\mathrm{bmit}a + \backslash\mathrm{bmit}b)\\ \overrightarrow{BM} &= -\overrightarrow{AB} + \overrightarrow{AM}\\ &= -\backslash\mathrm{bmit}b + \frac{1}{2}(\backslash\mathrm{bmit}a + \backslash\mathrm{bmit}b)\\ &= \frac{1}{2}(\backslash\mathrm{bmit}a - \backslash\mathrm{bmit}b)\\ \overrightarrow{MD} &= -\overrightarrow{AM} + \overrightarrow{AD}\\ &= -\frac{1}{2}(\backslash\mathrm{bmit}a + \backslash\mathrm{bmit}b) + \backslash\mathrm{bmit}a\\ &= \frac{1}{2}(\backslash\mathrm{bmit}a - \backslash\mathrm{bmit}b)\\ &= \overrightarrow{BM}\end{aligned}$$

Thus M is the midpoint BD.

Therefore the diagonals of a parallelogram bisect each other.

10 Consider $\triangle ABC$. Let the altitudes from A to BC and B to AC meet at O.

Let $\overrightarrow{OA} = \backslash\mathrm{bmit}a, \overrightarrow{OB} = \backslash\mathrm{bmit}b$ and $\overrightarrow{OC} = \backslash\mathrm{bmit}c$.

Then

$(\backslash\mathrm{bmit}c - \backslash\mathrm{bmit}b)\cdot\backslash\mathrm{bmit}a = 0\ldots(1)$.

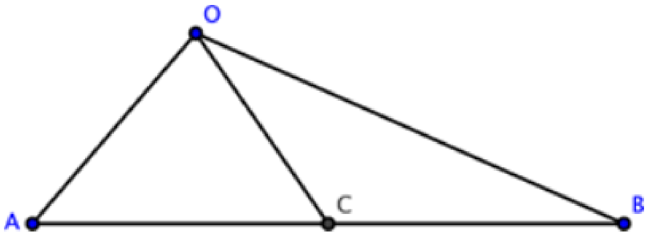
$(\backslash\mathrm{bmit}c - \backslash\mathrm{bmit}a)\cdot\backslash\mathrm{bmit}b = 0\ldots(2)$.

Subtract (1) from (2)

$$\begin{aligned}\backslash\mathrm{bmit}c - \backslash\mathrm{bmit}a)\cdot\backslash\mathrm{bmit}b - (\backslash\mathrm{bmit}c - \backslash\mathrm{bmit}b)\cdot\backslash\mathrm{bmit}a &= 0\\ \therefore \backslash\mathrm{bmit}c\cdot\backslash\mathrm{bmit}b - \backslash\mathrm{bmit}a\cdot\backslash\mathrm{bmit}b - \backslash\mathrm{bmit}c\cdot\backslash\mathrm{bmit}a + \backslash\mathrm{bmit}b\cdot\backslash\mathrm{bmit}a &= 0\\ \therefore \backslash\mathrm{bmit}c\cdot\backslash\mathrm{bmit}b - \backslash\mathrm{bmit}c\cdot\backslash\mathrm{bmit}a &= 0\\ \therefore \backslash\mathrm{bmit}c\cdot(\backslash\mathrm{bmit}b - \backslash\mathrm{bmit}a) &= 0\end{aligned}$$

Therefore OC is the altitude from C to AB

11

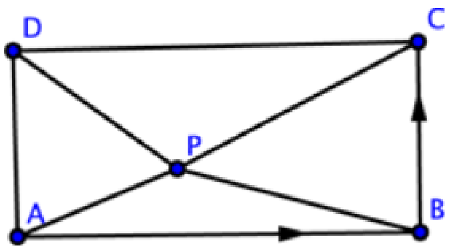


$$\overrightarrow{OC} = \overrightarrow{OA} + \frac{1}{2}(\overrightarrow{AO} + \overrightarrow{OB})$$
$$= \bmit a + \frac{1}{2}(-\bmit a + \bmit b)$$
$$= \frac{1}{2}(\bmit a + \bmit b)$$
$$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$$
$$= \frac{1}{2}(\bmit b - \bmit a)$$
$$4\overrightarrow{OC}\cdot\overrightarrow{OC} = \bmit b \cdot \bmit b + \bmit a \cdot \bmit a + 2\bmit b \cdot \bmit a$$
$$= |\bmit a|^2 + |\bmit b|^2 + 2\bmit b . \bmit a$$
$$4\overrightarrow{AC}\cdot\overrightarrow{AC} = |\bmit a|^2 + |\bmit b|^2 - 2\bmit b . \bmit a$$

Therefore

$$4|\overrightarrow{OC}|^2 + 4|\overrightarrow{AC}|^2 = 2|\bmit a|^2 + 2|\bmit b|^2$$
$$\therefore 2|\overrightarrow{OC}|^2 + 2|\overrightarrow{AC}|^2 = |\bmit a|^2 + |\bmit b|^2$$

12



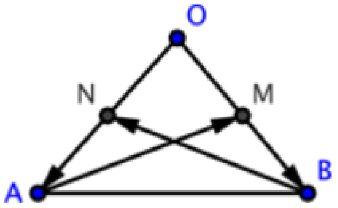
For rectangle $ABCD$

Let $\overrightarrow{AB} = \bmit x$ and $\overrightarrow{BC} = \bmit y$

Then there exist real numbers $0 < \lambda < 1$ and $0 < \mu < 1$ such that:

$$\overrightarrow{PB} = \lambda\bmit x + \mu\bmit y$$
$$\overrightarrow{PC} = \lambda\bmit x + (1-\mu)\bmit y$$
$$\overrightarrow{PD} = -(1-\lambda)\bmit x + (1-\mu)\bmit y$$
$$\overrightarrow{PA} = -(1-\lambda)\bmit x - \mu\bmit y$$
$$|\overrightarrow{PB}|^2 + |\overrightarrow{PD}|^2 = \lambda^2|\bmit x|^2 + \mu^2|\bmit y|^2 + (1-\lambda)^2|\bmit x|^2 + (1-\mu)^2|\bmit y|^2$$
$$|\overrightarrow{PA}|^2 + |\overrightarrow{PC}|^2 = (1-\lambda)^2|\bmit x|^2 + \mu^2|\bmit y|^2 + \lambda^2|\bmit x|^2 + (1-\mu)^2|\bmit y|^2$$
$$\therefore |\overrightarrow{PB}|^2 + |\overrightarrow{PD}^2| = |\overrightarrow{PA}|^2 + |\overrightarrow{PC}|^2$$

13



Let $OA = OB$

Let $\backslash\mathrm{bmit}a = \overrightarrow{OA}$ and $\backslash\mathrm{bmit}b = \overrightarrow{OB}$

Let M be the midpoint of OB and N be the midpoint of OA.

$$\overrightarrow{AM} = \overrightarrow{AO} + \frac{1}{2}\overrightarrow{OB}$$

$$= -\backslash\mathrm{bmit}a + \frac{1}{2}\backslash\mathrm{bmit}b$$

$$\overrightarrow{BN} = \overrightarrow{BO} + \frac{1}{2}\overrightarrow{OA}$$

$$= -\backslash\mathrm{bmit}b + \frac{1}{2}\backslash\mathrm{bmit}a$$

$$|\overrightarrow{AM}|^2 = (-\backslash\mathrm{bmit}a + \frac{1}{2}\backslash\mathrm{bmit}b) \cdot (-\backslash\mathrm{bmit}a + \frac{1}{2}\backslash\mathrm{bmit}b)$$

$$= \backslash\mathrm{bmit}a \cdot \backslash\mathrm{bmit}a - \backslash\mathrm{bmit}a \cdot \backslash\mathrm{bmit}b + \frac{1}{4}\backslash\mathrm{bmit}b \cdot \backslash\mathrm{bmit}b$$

$$= |\backslash\mathrm{bmit}a|^2 + \frac{1}{4}|\backslash\mathrm{bmit}b|^2$$

$$|\overrightarrow{BN}|^2 = (\frac{1}{2}\backslash\mathrm{bmit}a - \backslash\mathrm{bmit}b) \cdot (\frac{1}{2}\backslash\mathrm{bmit}a - \backslash\mathrm{bmit}b)$$

$$= \frac{1}{4}\backslash\mathrm{bmit}a \cdot \backslash\mathrm{bmit}a - \backslash\mathrm{bmit}a \cdot \backslash\mathrm{bmit}b + \backslash\mathrm{bmit}b \cdot \backslash\mathrm{bmit}b$$

$$= \frac{1}{4}|\backslash\mathrm{bmit}a|^2 + |\backslash\mathrm{bmit}b|^2$$

But $|\backslash\mathrm{bmit}a|| = |\backslash\mathrm{bmit}b|$.

Hence $|\overrightarrow{BN}| = |\overrightarrow{AM}|$

14 See question 10